Characterizing Sandstone Rock Properties Using Data Analytics with Vipulanandan Failure and Correlation Models

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ABSTRACT

With the increasing construction activities in various types of rocks around the world related to transportation tunnels, support bridges, buildings, and storage facilities in sandstone, sandstone, and limestone, and also hydraulic fracking. In this study, over 320 data were used to characterize the sandstone rock behaviors including the statistical distributions of the data. The study included density, pulse velocity, permeability, and strength. The density of sandstone was in the range of 2.17 to 2.74 g/cm^3 . The compressive strength and tensile strength of the rocks investigated varied up to 145 MPa and 22 MPa respectively. Vipulanandan correlation model was effective in relating the tensile strengths to the compressive strength of the rocks. At present, Mohr-Coulomb criteria are used to characterize the failure of the rocks and it overestimates the tensile strength and has no limit on the maximum shear strength tolerance for the rocks. The new Vipulanandan failure model was developed to not only better quantify the tensile strength but also to predict the maximum shear strength tolerance of the rocks The maximum shear stress (τmax.) represeanting the shar strength for the sandstone rock was 103 MPa.

1. Introduction

It is necessary to better quantify the mechanical properties and failure criteria of the rocks to construct different types of infrastructure on the rocks (Mohammed and Mahmood, 2018 a). It is also important to design the drilling and fracturing processes using the rock properties, including the tensile strength and failure criteria, for hydraulic fracturing of rocks in a cost-effective way (Mohammed and Mahmood, 2018 b). To define the failure of the rocks, Mohr-Coulomb criteria is used with over prediction of the tensile strength and no limit on the maximum shear strength tolerance with the normal stress on the rocks applied (Singh et al., 2015; Vipulanandan and Mohammed, 2018; Mahmood et al., 2020). Hence, there is a need to developing property correlations and improved the failure criteria of rocks since, there is very limited property correlation in the literature (Omar, 2017).

Sandstone is a fine-grained sedimentary rock composed of mud that is a mix of flakes of clay minerals and tiny fragments (silt-sized particles) of other minerals, especially quartz and calcite (Tucker 2009). In rock engineering, most applied rock classification systems are based on mechanical parameters such as uniaxial unconfined compressive strength (UCS), tensile strength (σt), and Young's modulus or deformability modulus (E) (Selçuk and Nar 2016). Unconfined compressive strength (UCS), the most widely used property to evaluate rock strength, costly and time-consuming testing with sample preparation and testing (Karakus et al. 2005). Many researchers have introduced several empirical equations for the determination of rock strength from simple physical properties. Using such properties, rock strength may be determined in an easy, quick, and inexpensive manner during field investigations (Sabatakakis et al. 2008, Rajabzadeh et al. 2012). Tensile strength and fracture toughness are two important parameters in rock mechanics, and it is used in the initiation and propagation of fractures in hydraulic fracture modeling (Meng and Pan 2007, Vipulanandan and Mohammed 2014).

2. Objectives

Quantify the mechanical behavior of the sandstone rocks based on more than 400 data collected from previous research studies are the objective of this study. The main objectives are as follows:

1. To qualify the statistical variation in the density, compressive strength, and tensile strength of sandstone rocks.
2. To investigate and quantify the correlation relationship between the compression strength and tensile strength and fracture toughness of sandstone rocks using the Vipulanandan correlation model.
3. Quantify the shear failure strength for the sandstone rock using the new Vipulanandan failure model.

3. Methods and materials

a. Data collection

This study focused on the behavior of sandstone of rocks based on the data collected from several research studies. The properties of interest were density, compressive and tensile strengths, fracture toughness of limestone, sandstone, and sandstone rocks.

b. Modeling

Vipulanandan correlation model

The correlation between the properties of sandstone rock was investigated using the Vipulanandan correlation model. Based on the inspection of the data collected the following relationship was selected (Mohammed and Vipulanandan, 2014 & 2015; Mohammed and Mahmood, 2019; Mahmood and Mohammed, 2020):

$$\mathrm{Y} = \mathrm{Yo} + \frac{\mathrm{X}}{(\mathrm{A}+\mathrm{B}*\mathrm{X})} \tag{1}$$

where:
Y= tensile strength (depended variables).
A and B = model parameters depend on the rock properties and environments.
X= compression strength (in-depended variables).

Vipulanandan failure model

Based on years of experience and reviewing the material shear strength versus applied normal stress is nonlinear, also there is a limit to the maximum shear stress tolerance for all the materials, and hence following model and conditions are proposed (Vipulanandan and Mohammed, 2014):

$$\tau = \tau_o + \frac{\sigma_n}{C+D*\sigma_n} \tag{2}$$

when $\sigma_n \rightarrow \infty$

$$\tau_\infty = \tau_o + \frac{1}{D} \tag{3}$$

Hence, this model (Eq.3) has a limit on the maximum shear stress the rocks will tolerate at relatively high normal stress.

4. Result and analysis

Statistical analysis

(i). Density (γ)

Based on the 184 data collected on the sandstone rock from the literature (Table 1), the median values of density was 2.49 gm/cm^3 as shown in Figure 1. The density of sandstone rock varied from 2.17 gm/cm^3 to 2.74 gm/cm^3 based on 60 data with the standard deveiation (SD) of 0.158 gm/cm^3 and variance of 0.025 as summarized in Table 1. For the density of sandstone rock, Normal distribution was selected based on the AD and P-value testing (Figure 1).

Table 1. Statistical parameters of geotechnical properties of rocks

	Statistical Parameters	**Density (gm/cm^3)**	**Compressive Strength, σ_c (MPa)**	**Tensile Strength, σ_t (MPa)**
Sandstone	No. of Data	60	184	84
	Range	2.17-2.74	19-145	0.12-22
	Mean (μ)	2.49	86.15	8.68
	Std. Deviation (σ)	0.158	32.9	5.22
	Variance	0.025	1083.03	27.27

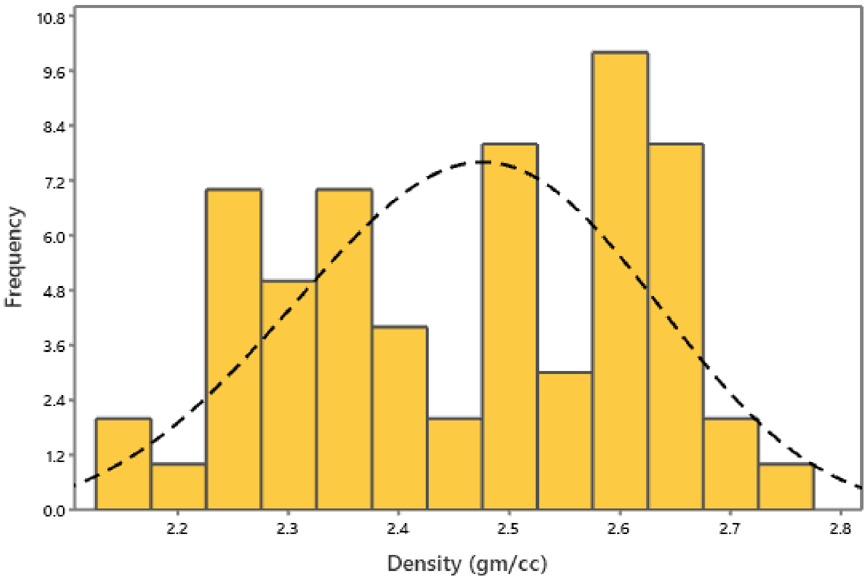

Figure 1. Statistical Varaition of the Density of the Sandstone Rock

Compressive strength (σ_c):

Based on 184 of σ_c data for sandstone rock, the data varied from 19 MPa to 145 MPa with a median of 86.15 MPa, the SD of 32.9 MPa, and variance of 1083 as summarized in Table 1. Based on AD and P-value testing, normal distribution for the σ_c for the sandstone rock was selected as shown in Figure 2.

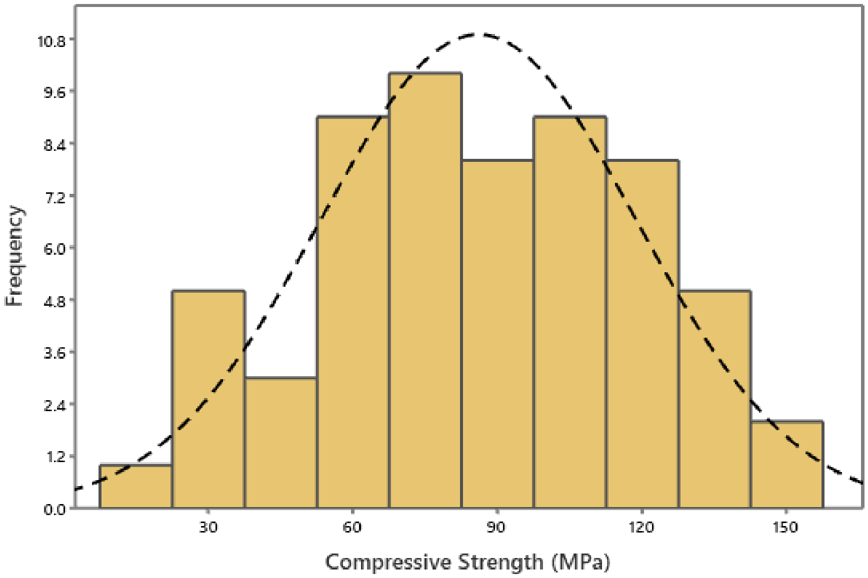

Figure 2. Statistical Varaition of the Compressive Strength of the Sandstone Rocks

Tensile strength (σ_t)

Based on a total of 84 tensile strength data for sandstone, the data varied from 0.12 MPa to 22 MPa with a median of 8.68 MPa, the SD of 5.22 MPa. Based on the AD and P-value testing, 3-Parameter Weibull distribution for the σ_t of sandstone was selected as shown in Figure 3.

Property correlation

(i). Compressive strength (σ_c) and Density (γ)

Based on more than 180 data of the sandstone rocks collected from the literature (Table 1), no direct correlation was observed between the density (γ) and compressive strength (σ_c) as shown in Figure 4.

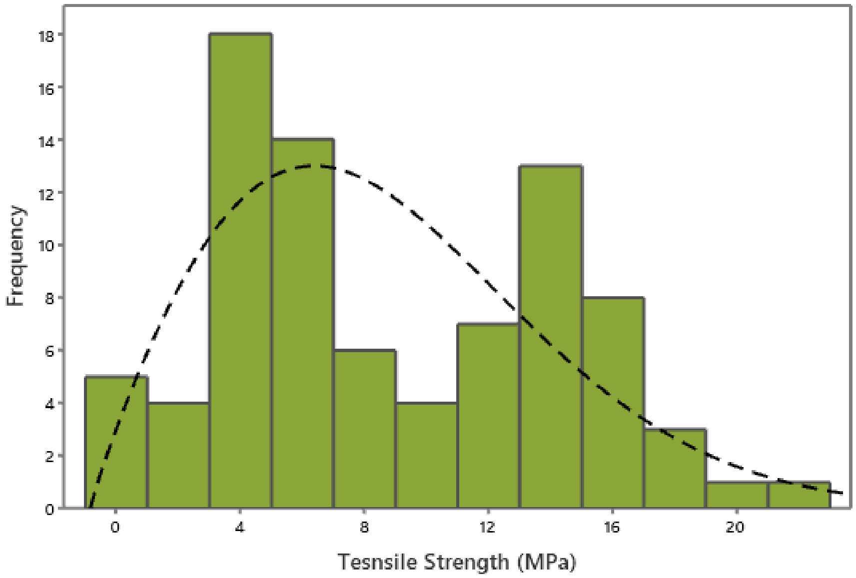

Figure 3. Statistical Varaition of the Tensile Strength of the Sandstone Rocks

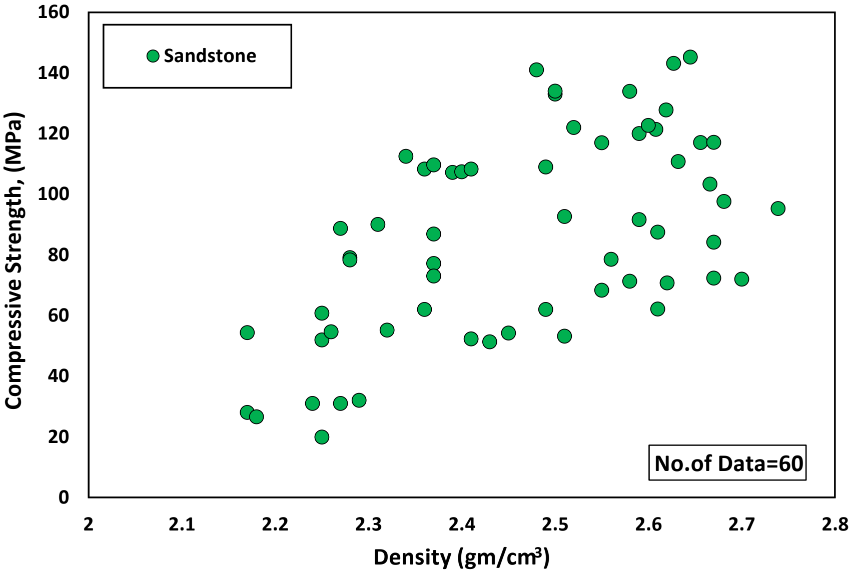

Figure 4. Varaition of the Compressive Strength with the Density for the Sandstone Rocks

(ii). Tensile strength (σ_t) and compressive strength (σ_c)

A total of 23 data were collected from various research studies. With the increasing of σ_c of rocks, the σ_t nonlinearly also increased as shown in Figure 5. The change in the σ_c with σ_t of rocks was represented using the Vipulanandan correlation model relationship (Eq. 1) and the model parameters A, B, coefficient of determination (R^2), and root mean square error (RMSE) were 8.6, 0.046 MPa^{-1}, 0.84 and 1.31 MPa respectively as summarized in Table 2. With the increase in σ_c of the sandstone rock from 10 MPa to 100 MPa, the σ_t increased from 1.8 MPa to 11 MPa (Fig. 5).

Table 2. Model parameters for tensile and compression strength relationship

			Vipulanandan correlation model (Eq.1)				
Dependend Variable (Y-axis)	**In depended Variable (X-axis)**	**Type of Rock**	**A**	**B MPa^{-1}**	**RMSE (MPa)**	**R^2**	**No. of Data**
Tensile Strength, σt (MPa)	**Compressive Strength, σc (MPa)**	**Sandstone**	8.6	0.046	1.31	0.84	23

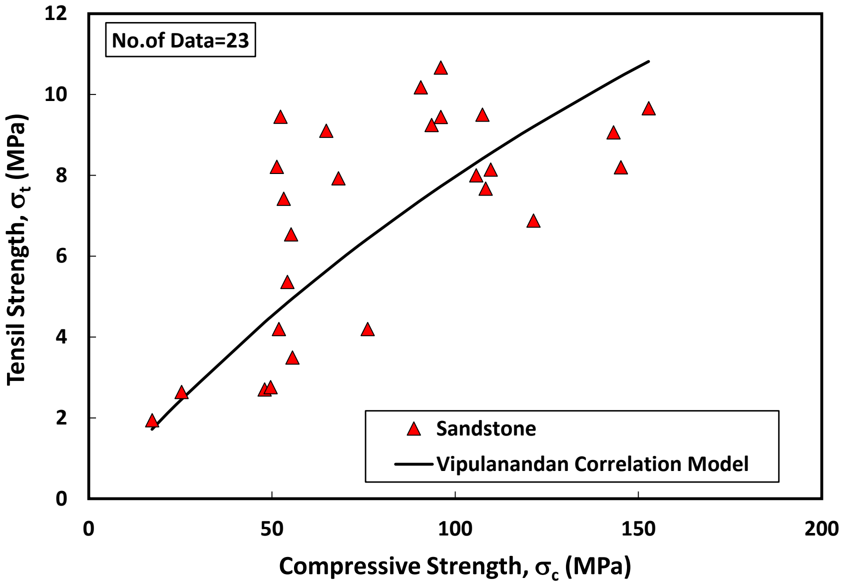

Figure 5. Varaition of the Tensile Strength with the Compressive Strength for the Sandstone Rocks

(iii). Failure Model

Vipulanandan failure model

The shear stress behavior of sandstone rock with the 55 data collected from the literature was analyzed using the Vipulanandan failure model (Eq. (2)). The coefficient of determination (R^2) and root mean square of error (RMSE) were 0.95 and 2.6 MPa respectively as summarized in Table 3. The yield stress (τ_o) and tensile strength (σ_t) of the sandstone were 3.0 MPa and 3.9 MPa respectively as summarized in Table 3. The model parameters C and D for sandstone were 1.34 and 0.01 MPa^{-1} respectively as summarized in Table 3.

Maximum shear strength ($\tau_{max.}$)

Based on Eqn. 3, the Vipulanandan failure model has a limit on the maximum shear stress the rocks will tolerate. Based on the limited data, the τ_{max} for sandstone rock was 103 MPa as summarized in Table 3.

Table 3. Failure Model Parameters for the Sandstone Rock

Vipulanandan failure model (Eq.3)							
τ_o (MPa)	σ_t (MPa)	C	D MPa^{-1}	$\tau_{max.}$ (MPa) (Eq. 9)	RMSE (MPa)	R^2	No. of Data
3	3.9	1.34	0.010	103	2.6	0.95	55

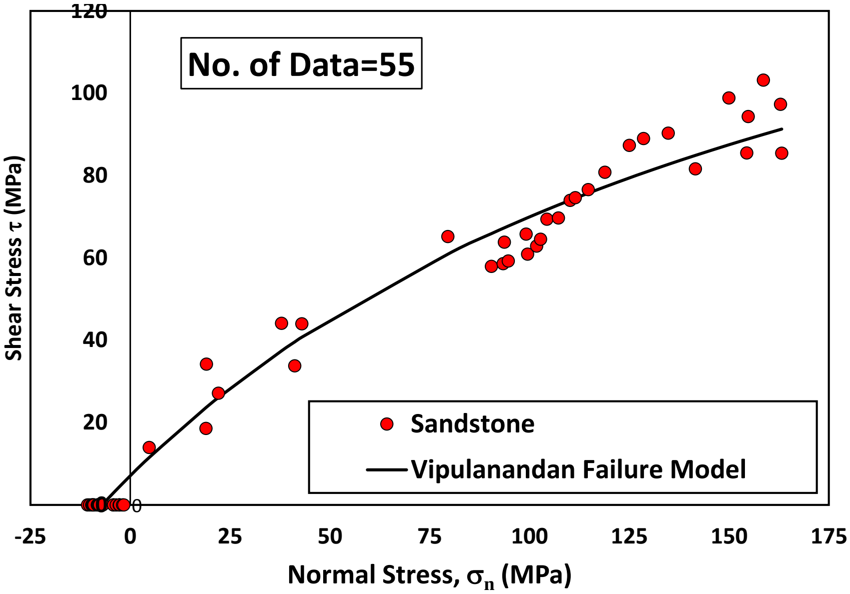

Figure 6. Comparing the Failure Data with the Vipulanandan Failure Model Predicton for the Sandstone Rock

5. Conclusions

The focus of this study was to characterize the sandstone rock based on the density and strength properties based on over 300 data collected and the following conclusions were advanced:

1. The compressive strength (σ_c) of the sandstone rock varied 2 to 200 MPa respectively. Based on the statistical analysis the median values of density was 2.50 gm/cm^3 respectively.
2. The tensile strength to compressive strength ratio for the sandstone rock varied from 0.05 to 0.2 compared to 0.1 for concrete.
3. The Vipulanandan correlation model was effective in predicting the relationship between tensile strength and compressive strength of the sandstone rock.

4. The new Vipulanandan failure criterion has been developed to not only better quantify the shear stress but also maximum shear strength tolerance of the rocks. The maximum shear stress ($\tau_{max.}$) for sandstone was 103 MPa respectively.

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