Characterizing Shale Rock Properties Using Data Analytics with Vipulanandan Failure and Correlation Models

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Abstract
With the increasing construction activities in various types of rocks around the world related to transportation tunnels, support bridges, buildings, and storage facilities in sandstone, shale, and limestone, and also hydraulic fracking. In this study, over 400 data were used to characterize the shale rock behaviors including the statistical distributions of the data. The study included density, pulse velocity, permeability, and strength. The density of shale was in the range of 1.7 to 3.0 g/cm^3 . The compressive strength and tensile strength of the rocks investigated varied up to 101 MPa and 9.5 MPa respectively. Vipulanandan correlation model was effective in relating the tensile strengths, permeability, and ultrasonic pulse velocity to the compressive strength of the rocks. At present, Mohr-Coulomb criteria are used to characterize the failure of the rocks and it overestimates the tensile strength and has no limit on the maximum shear strength tolerance for the rocks. The new Vipulanandan failure model was developed to not only better quantify the tensile strength but also to predict the maximum shear strength tolerance of the rocks The maximum shear strength (τ_{max}.) for the shale rock was 102 MPa.

1. Introduction

It is necessary to better quantify the mechanical properties and failure criteria of the rocks to construct different types of infrastructure on the rocks (Mohammed and Mahmood, 2018 a). It is also important to design the drilling and fracturing processes using the rock properties, including the tensile strength and failure criteria, for hydraulic fracturing of rocks in a cost-effective way (Mohammed and Mahmood, 2018 b). To define the failure of the rocks, Mohr-Coulomb criteria is used with over prediction of the tensile strength and no limit on the maximum shear strength tolerance with the normal stress on the rocks applied (Singh et al., 2015; Vipulanandan and Mohammed, 2018; Mahmood et al., 2020). Hence, there is a need to developing property correlations and improved the failure criteria of rocks since, there is very limited property correlation in the literature (Omar, 2017).

Shale is a fine-grained sedimentary rock composed of mud that is a mix of flakes of clay minerals and tiny fragments (silt-sized particles) of other minerals, especially quartz and calcite (Tucker 2009). In rock engineering, most applied rock classification systems are based on mechanical parameters such as uniaxial unconfined compressive strength (UCS), tensile strength (σt), and Young's modulus or deformability modulus (E) (Selçuk and Nar 2016). Unconfined compressive strength (UCS), the most widely used property to evaluate rock strength, costly and time-consuming testing with sample preparation and testing (Karakus et al. 2005). Many researchers have introduced several empirical equations for the determination of rock strength from simple physical properties. Using such properties, rock strength may be determined in an easy, quick, and inexpensive manner during field investigations (Sabatakakis et al. 2008, Rajabzadeh et al. 2012). Tensile strength and fracture toughness are two important parameters in rock mechanics, and it is used in the initiation and propagation of fractures in hydraulic fracture modeling (Meng and Pan 2007, Vipulanandan and Mohammed 2014).

2. Objectives

Quantify the mechanical behavior of the shale rocks based on more than 100 data collected from previous research studies are the objective of this study. The main objectives are as follows:

1. To qualify the statistical variation in the density, compressive strength, and tensile strength of shale rocks.
2. To investigate and quantify the correlation relationship between the compression strength and tensile strength and fracture toughness of shale rocks using the Vipulanandan correlation model.
3. Quantify the shear failure strength for the rocks using the new Vipulanandan failure model.

3. Methods and materials

a. Data collection

This study focused on the behavior of shale of rocks based on the data collected from several research studies. The properties of interest were density, compressive and tensile strengths, fracture toughness of limestone, sandstone, and shale rocks.

b. Modeling

Vipulanandan correlation model

The correlation between the properties of shale rock was investigated using the Vipulanandan correlation model. Based on the inspection of the data collected the following relationship was selected (Mohammed and Vipulanandan, 2014 & 2015; Mohammed and Mahmood, 2019; Mahmood and Mohammed, 2020):

$$\mathrm{Y} = \mathrm{Yo} + \frac{\mathrm{X}}{(\mathrm{A}+\mathrm{B}*\mathrm{X})} \tag{1}$$

where:
Y= tensile strength (depended variables).
A and B = model parameters depend on the rock properties and environments.
X= compression strength (in-depended variables).

Vipulanandan failure model

Based on years of experience and reviewing the material shear strength versus applied normal stress is nonlinear, also there is a limit to the maximum shear stress tolerance for all the materials, and hence following model and conditions are proposed (Vipulanandan and Mohammed, 2014):

$$\tau = \tau_o + \frac{\sigma_n}{C+D*\sigma_n} \tag{2}$$

when $\sigma_n \rightarrow \infty$

$$\tau_\infty = \tau_o + \frac{1}{D} \tag{3}$$

Hence, this model (Eq.3) has a limit on the maximum shear stress the rocks will tolerate at relatively high normal stress.

4. Result and analysis

Statistical analysis

(i). Density (γ)

Based on the 184 data collected on the shale rock from the literature (Table 1), the median values of density was 2.50 gm/cm^3 as shown in Figure 1. The density of shale rock varied from 1.70 gm/cm^3 to 3 gm/cm^3 based on 184 data with the standard deviation (SD) of 0.248 gm/cm^3 and variance of 0.061 as summarized in Table 1. For the density of shale rock, Weibull distribution was selected based on the AD and P-value testing (Figure 1).

Table 1. Statistical parameters of geotechnical properties of rocks

	Statistical Parameters	Density (gm/cm^3)	Compressive Strength, σ_c (MPa)	Tensile Strength, σ_t (MPa)
Shale	No. of Data	184	184	81
	Range	1.7-3	2-200	2.5-9.5
	Mean (μ)	2.5	61.90	6.8
	Std. Deviation (σ)	0.248	35.28	1.156
	Variance	0.061	1244.99	1.337

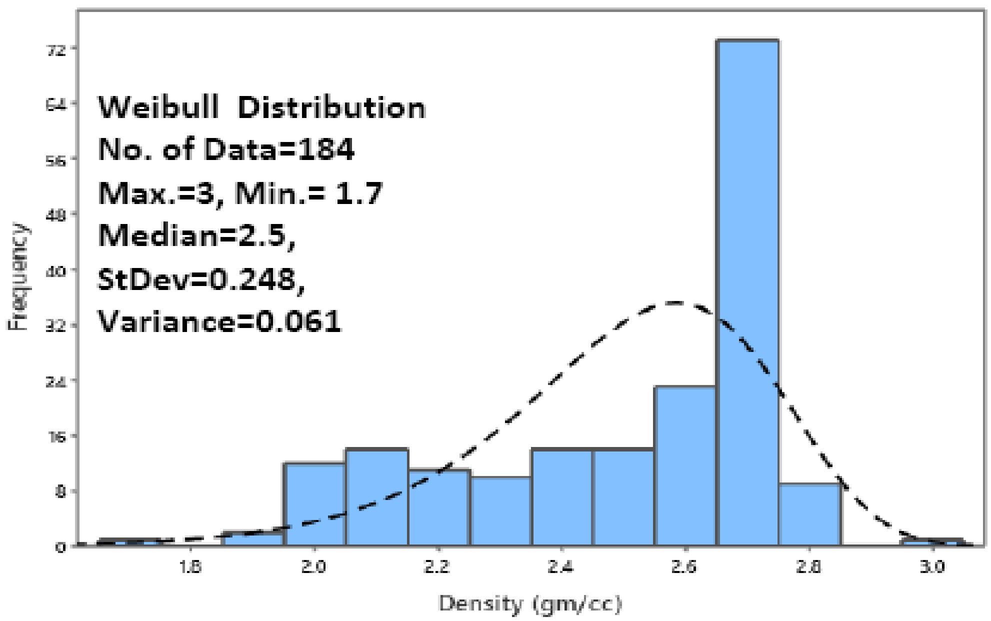

Figure 1. Statistical Varaition of the Density of the Shale Rock

Compressive strength (σ_c):

Based on 184 of σ_c data for shale rock, the data varied from 2 MPa to 200 MPa with a median of 61.9 MPa, the SD of 35.28 MPa, and variance of 1245 as summarized in Table 1. Based on AD and P-value testing, normal distribution for the σ_c for the shale rock was selected as shown in Figure 2.

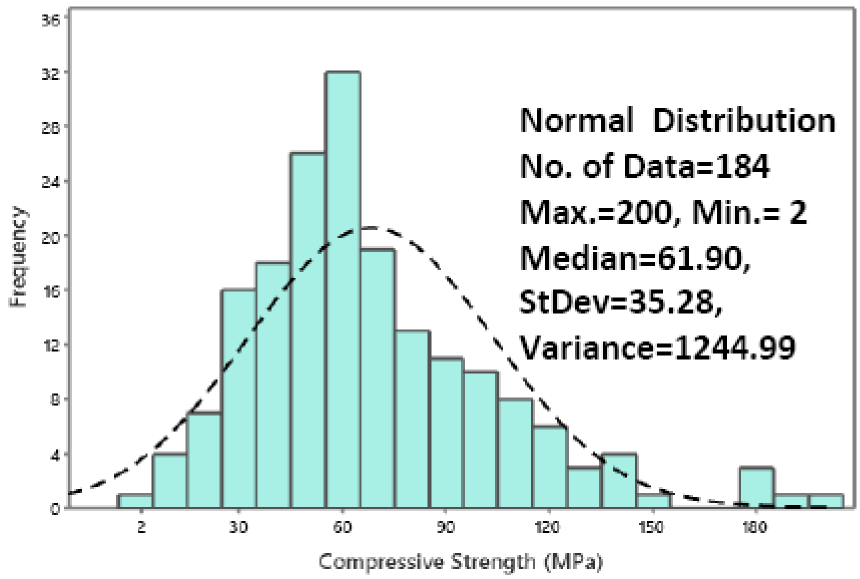

Figure 2. Statistical Varaition of the Compressive Strength of the Shale Rocks

Tensile strength (σ_t)

Based on a total of 81 tensile strength data for shale, the data varied from 2.5 MPa to 9.5 MPa with a median of 6.8 MPa, the SD of 1.156 MPa. Based on the AD and P-value testing, 3-Parameter Weibull for the σ_t of shale was selected as shown in Figure 3.

Property correlation

(i). Compressive strength (σ_c) and Density (γ)

Based on more than 180 data of the shale rocks collected from the literature (Table 1), no direct correlation was observed between the density (γ) and compressive strength (σ_c) as shown in Figure 4.

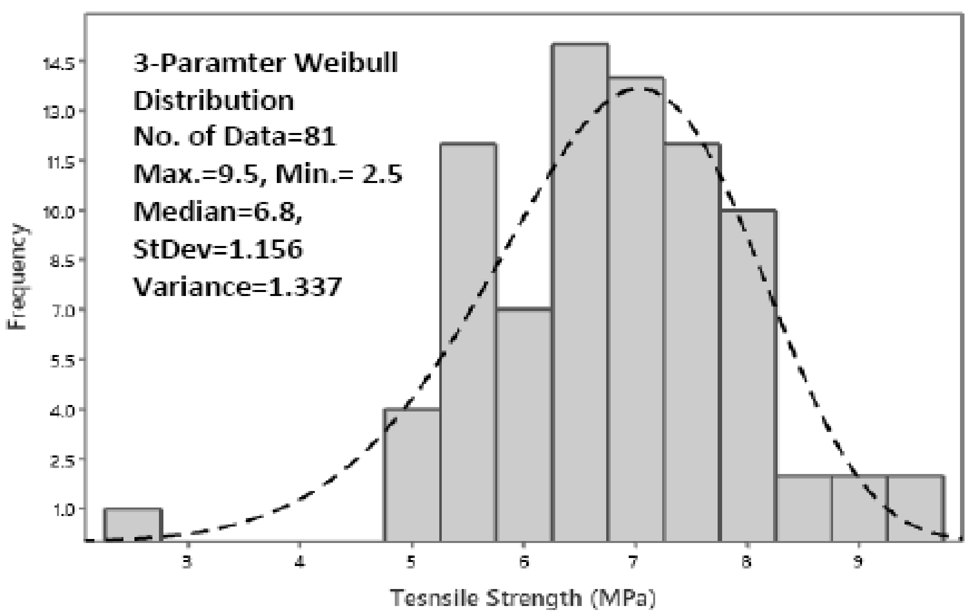

Figure 3. Statistical Variation of the Tensile Strength of the Shale Rocks

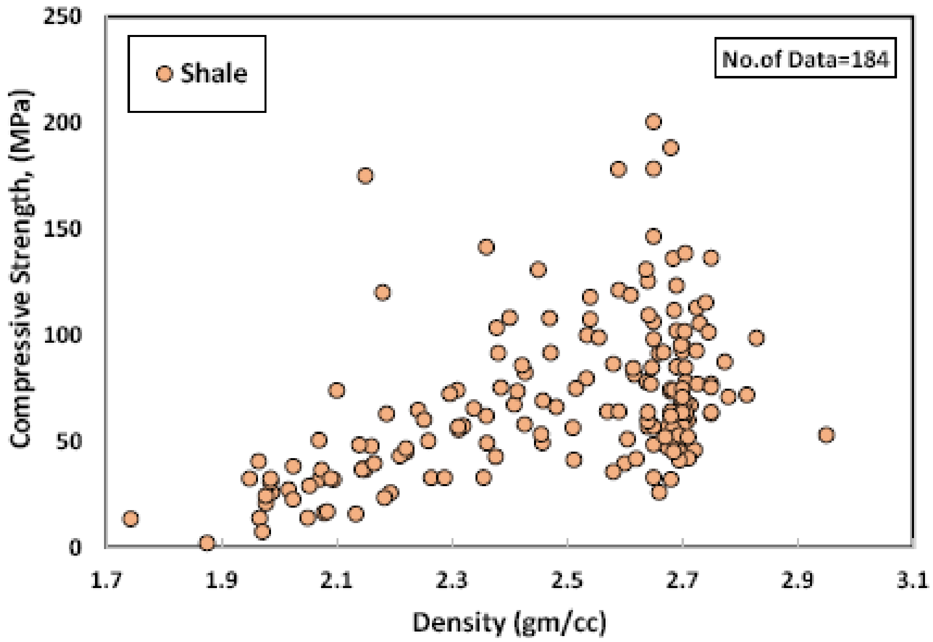

Figure 4. Varaition of the Compressive Strength with the Density for the Shale Rocks

(ii). Tensile strength (σ_t) and compressive strength (σ_c)

A total of 73 data were collected from various research studies. With the increasing of σ_c of rocks, the σ_t nonlinearly also increased as shown in Figure 5. The change in the σ_c with σ_t of rocks was represented using the Vipulanandan correlation model relationship (Eq. 1) and the model parameters A, B, coefficient of determination (R^2), and root mean square error (RMSE) were 8.2, 0.05 MPa^{-1}, 0.80 and 1.03 MPa respectively as summarized in Table 2. With the increase in σ_c of the shale rock from 10 MPa to 100 MPa, the σ_t increased from 1.8 MPa to 12 MPa (Fig. 5).

Table 2. Model parameters for tensile and compression strength relationship

			Vipulanandan correlation model (Eq.1)				
Depended Variable (Y-axis)	**In depended Variable (X-axis)**	**Type of Rock**	**A**	**B MPa^{-1}**	**RMSE (MPa)**	**R^2**	**No. of Data**
Tensile Strength, σt (MPa)	**Compressive Strength, σc (MPa)**	Shale	6.5	0.075	0.32	0.97	73

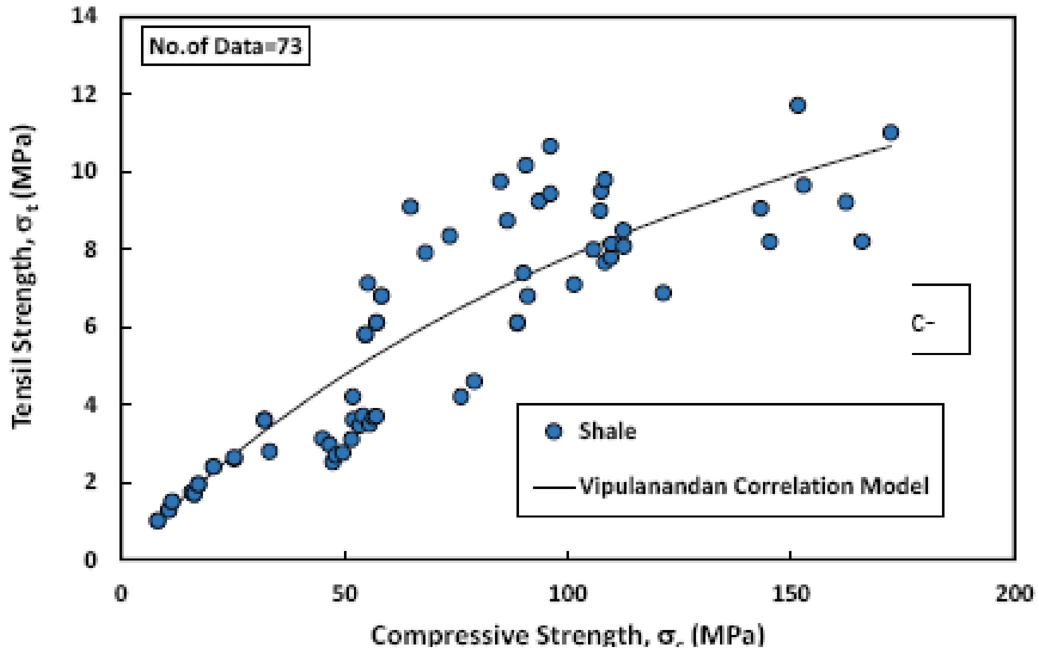

Figure 5. Varaition of the Tensile Strength with the Compressive Strength for the Shale Rocks

(iii). Failure Model

Vipulanandan failure model

The shear stress behavior of shale rock with the 29 data collected from the literature was analyzed using the Vipulanandan failure model (Eq. (2)). The coefficient of determination (R^2) and root mean square of error (RMSE) were 0.94 and 3.4 MPa respectively as summarized in Table 3. The yield stress (τ_o) and tensile strength (σ_t) of the shale were 2.6 MPa and 3.9 MPa respectively as summarized in Table 3. The model parameters C and D for shale were 1.26 and 0.01 MPa^{-1} respectively as summarized in Table 3.

Maximum shear strength ($\tau_{max.}$)

Based on Eqn. 3, the Vipulanandan failure model has a limit on the maximum shear stress the rocks will tolerate. Based on the limited data, the τ_{max} for shale rock was 102 MPa as summarized in Table 3.

Table 3. Failure Model Parameters for the Shale Rock

Vipulanandan failure model (Eq.3)							
τ_o (MPa)	σ_t (MPa)	C	D MPa^{-1}	$\tau_{max.}$ (MPa) (Eq. 9)	RMSE (MPa)	R^2	No. of Data
2.6	3.2	1.26	0.010	102	3.4	0.94	29

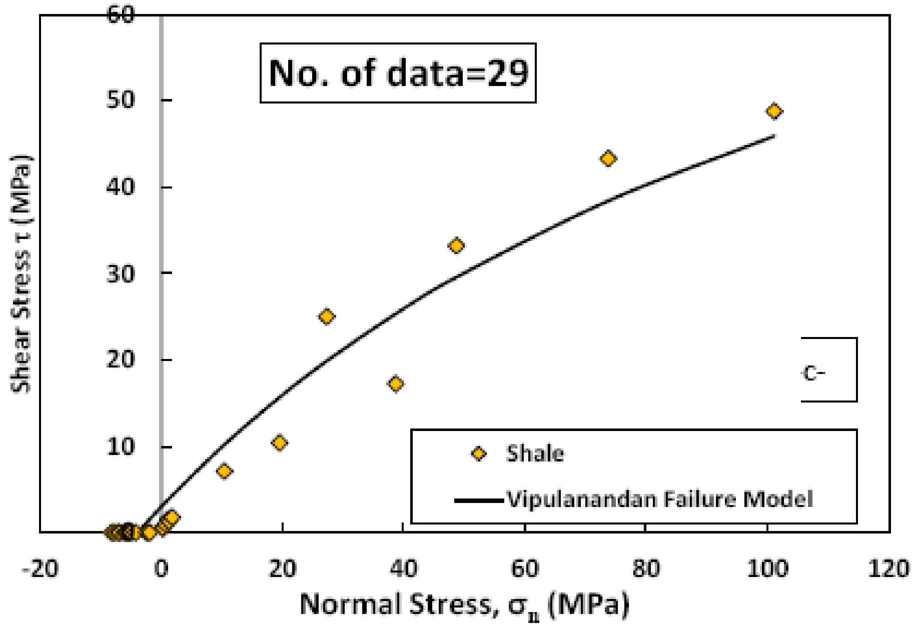

Figure 6. Comparing the Failure Data with the Vipulanandan Failure Model Prediction for the Shale Rock

5. Conclusions

The focus of this study was to characterize the shale rock based on the density and strength properties based on over 400 data collected and the following conclusions were advanced:

1. The compressive strength (σ_c) of the shale rock varied 2 to 200 MPa respectively. Based on the statistical analysis the median values of density was 2.50 gm/cm^3 respectively.
2. The tensile strength to compressive strength ratio for the shale rock varied from 0.05 to 0.2 compared to 0.1 for concrete.
3. The Vipulanandan correlation model was effective in predicting the relationship between tensile strength and compressive strength of the shale rock.
4. The new Vipulanandan failure criterion has been developed to not only better quantify the shear stress but also maximum shear strength tolerance of the rocks. The maximum shear stress ($\square_{max.}$) for shale was 102 MPa respectively.

6. Acknowledgements

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7. References

1. Abdelaali, R., B. Abderrahim, B. Mohamed, G. Yves, S. Abderrahim, H. Mimoun, and S. Jamal. 2013. “Prediction of Porosity and Density of Calcarenite Rocks from P-wave Velocity Measurements.” International Journal of Geosciences 4 (9). doi:10.4236/ijg.2013.49124

2. Karakus, M., Kumral, M., and Kilic, O. (2005). Predicting elastic properties of intact rocks from index tests using multiple regression modelling. International journal of rock mechanics and mining sciences (1997), 42(2), 323-330.
3. Mahmood, W., and Mohammed, A. (2020). New Vipulanandan pq model for particle size distribution and groutability limits for sandy soils. Journal of Testing and Evaluation, 48(5).
4. Mahmood, W., Mohammed, A., and Hama Hussein, S. (2020). Predicting mechanical properties and ultimate shear strength of gypsum, limestone and sandstone rocks using Vipulanandan models. Geomechanics and Geoengineering, 15(2), 90-106.
5. Mohammed, A. S., and Vipulanandan, C. (2014). Compressive and tensile behavior of polymer treated sulfate contaminated CL soil. Geotech. and Geology Engineering, 32(1), 71-83.
6. Mohammed, A. S., and Vipulanandan, C. (2014). Compressive and tensile behavior of polymer treated sulfate contaminated CL soil. Geotech. and Geology Engineering, 32(1), 71-83.
7. Mohammed, A., and Mahmood, W. (2018 a). Vipulanandan failure models to predict the tensile strength, compressive modulus, fracture toughness and ultimate shear strength of calcium rocks. International Journal of Geotechnical Engineering, 1-11.
8. Mohammed, A., and Mahmood, W. (2018 b). Statistical variations and new correlation models to predict the mechanical behavior and ultimate shear strength of gypsum rock. Open Engineering, 8(1), 213-226.
9. Mohammed, A., and Mahmood, W. (2019). Estimating the efficiency of the sandy soils-cement based grout interactions from particle size distribution (PSD). Geomechanics and Geoengineering, 1-18.
10. Mohammed, A., and Vipulanandan, C. (2015). Testing and modeling the short-term behavior of Lime and Fly Ash treated sulfate contaminated CL soil. Geotechnical and Geological Engineering, 33(4), 1099-1114.
11. Omar, M. (2017). Empirical correlations for predicting strength properties of rocks–United Arab Emirates. International Journal of Geotechnical Engineering, 11(3), 248-261.
12. Pettijohn, F.J. (2002): Sedimentary Rocks, Third dition, CBS Publishers and Distributors, 4596/1-A, 11 Daryaganj, New Delhi, - 110002 India, 628pp. Excavatability of Rock, Quarterly Journal of Engineering Geology, 27, pg 145-164
13. Rajabzadeh, M. A., Moosavinasab, Z., and Rakhshandehroo, G. (2012). Effects of rock classes and porosity on the relation between uniaxial compressive strength and some rock properties for carbonate rocks. Rock mechanics and rock engineering, 45(1), 113-122.
14. Sabatakakis, N., Koukis, G., Tsiambaos, G., and Papanakli, S. (2008). Index properties and strength variation controlled by microstructure for sedimentary rocks. Engineering Geology, 97(1-2), 80-90.
15. Singh, M., Samadhiya, N. K., Kumar, A., Kumar, V., and Singh, B. (2015). A nonlinear criterion for triaxial strength of inherently anisotropic rocks. Rock Mechanics and Rock Engineering, 48(4), 1387-1405.
16. Tucker, M. E. (Ed.). (2009). Sedimentary petrology: an introduction to the origin of sedimentary rocks. John Wiley and Sons
17. Vipulanandan, C. and Mohammed, A. (2018). New Vipulanandan failure model and property correlations for sandstone, shale and limestone rocks. ASCE, GSP 296, pp. 365-376.
18. Vipulanandan, C., and Mohammed, A. S. (2014). Hyperbolic rheological model with shear stress limit for acrylamide polymer modified bentonite-drilling muds. Journal of Petroleum Science and Engineering, 122, 38-47.
19. Vipulanandan, C., Mohammed, A., and Qu, Q. (2014). Characterizing the hydraulic fracturing fluid modified with nano silica proppant. American Association of Drilling Engineers (AADE) 2014, AADE-14-NTCE-33.