Evaluating Axial Elastic Pile Response From Cone Penetration Tests

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ABSTRACT: Axial pile performance can be rationally evaluated within an elastic continuum framework using field results from seismic piezocone tests (SCPTu). Using a versatile Randolph-type elastic pile model, the approach can be applied to either traditional top down loading using an anchored reaction beam or the newer Osterberg cell that simultaneously pushes the base and shaft in opposite directions. The axial load distribution within the shaft is also evaluated. For site-specific data at a given site, the SCPTu is an optimal means for collection of subsurface information because it combines penetrometer readings and downhole geophysics in one sounding. The results obtained are at opposite ends of the stress-strain-strength curves, specifically the peak strength for capacity interpretations and the small-strain stiffness \((E_{\text{max}})\) for evaluating the initial deformations. Axial pile capacity can be analysed using both direct and indirect CPT methods. Case studies are presented for deep foundations situated in stiff clays at two national geotechnical test sites located in Houston and College Station, Texas, using top down loading, as well as a third case study of a drilled shaft in clay till loaded by O-cell in Alberta.

INTRODUCTION

The axial load-displacement response of pile foundations is conveniently and logically represented within the context of an elastic continuum analysis, where the stiffness of the soil medium is expressed as an equivalent Young's modulus \(E_s\) and Poisson's ratio \(\nu\) (Poulos & Davis, 1980). For the simple case of a homogeneous soil medium (i.e., \(E_s\) and \(\nu\) are constant with depth), the top displacement \((w_t)\) of an embedded pile having a length \(L\) and diameter \(d\) that is subjected to an applied axial force \(Q_t\) (also commonly designated as \(P_t\)) is given by:

\[
 w_t = \frac{Q_t \cdot I_p}{d \cdot E_s} \tag{1}
\]
where $I_p$ = displacement influence factor. For rigid piles, the value of $I_p$ depends simply upon the slenderness ratio ($L/d$) and $\nu$, as indicated by the closed-form solution (Randolph & Wroth, 1978, 1979):

$$I_p = \frac{1}{1 - \nu^2 + \frac{\pi}{(1 + \nu)} \cdot \ln[5(L/d)(1-\nu)]}$$

Higher order equations can capture more complex features including: an underlying hard layer beneath the pile toe, pier with a belled base, soil stiffness increasing along the pile sides (i.e., Gibson soil), and pile compressibility (Poulos, 1989; Fleming, et al. 1992). For instance, the case of a pile embedded within a finite layer Gibson soil with the pile tip resting on a stiffer stratum is depicted in Figure 1. A generalized Gibson soil has the equivalent Young's modulus $E_s$ increasing linearly with depth:

$$E_s = E_{s0} + k_E z$$

where $E_{s0}$ = soil modulus at the ground surface, $z$ = depth, and $k_E = \Delta E_s/\Delta z =$ modulus rate parameter. In this case, the characteristic soil modulus for (1) is taken as that value along the sides at the tip (e.g., $E_{sL}$). The geomaterial stiffness beneath the pile tip/toe is designated as $E_b$ and may be same (floating pile) or different (end-bearing).

Figure 1. General simplified soil model for elastic pile foundation in two-layer system.
The solution for the load-displacement relationship of a rigid pile in a two-layer soil system is presented in Figure 2. In addition to top displacement, the solution gives the proportion of load transmitted to the pile tip/toe/base ($P_b/P_t$). In this arrangement, the nonhomogeneity of the modulus increasing with depth is represented by the parameter rho, which is defined as the mid-length modulus to that value at the pile full length: $\rho_E = E_{sm}/E_{sl}$. As these analytical solutions are closed-form, they have been termed the Randolph-type pile model (Randolph & Wroth, 1978, 1979).

AXIAL CAPACITY OF DEEP FOUNDATIONS

In geotechnical practice, the axial capacity of deep foundations is evaluated from methods based in static equilibrium, limit plasticity, and/or cavity expansion theory. Such solutions require the evaluation of soil engineering parameters, such as soil unit weight ($\gamma$), friction angle ($\phi'$), undrained shear strength ($s_u$), overconsolidation ratio (OCR), lateral stress coefficient ($K_0$), interface friction (tan\(\delta\)), and other variables (e.g., Kulhawy, et al. 1983; O'Neill & Reese, 1999). Methods for evaluating various soil parameters from a variety of in-situ field tests are given elsewhere, such as Kulhawy & Mayne (1990) and Schnaid (2009). Specific to the CPT and CPTu, detailed guides are given in Lunne et al. (1997) and Mayne (2007).
Alternatively, a number of direct in-situ methods have been developed in order to scale field results up from small penetrometers and/or probes to obtain a unit side friction and/or unit end bearing resistance for the large pile foundations. Direct methods have been proposed for the standard penetration test (SPT), cone (CPT), flat dilatometer (DMT), pressuremeter (PMT), and vane shear test (VST). For instance, Poulos (1989) reviews several approaches using SPT and/or CPT data. These methods have been developed empirically and are usually only applicable to a particular type of deep foundation (i.e., driven, drilled, jacked, vibrated, pressed) and specific geologic formation of concern (i.e., clay, sand, silt, residual soils, intermediate geomaterials).

In a few instances, generalized direct solutions for pile capacity evaluation have been attempted that apply to a number of different pile types in a variety of soil types. For the CPT, these include the well-known LCPC method (Bustamante & Gianeselli, 1982; Frank & Magnan, 1995; Bustamante & Frank, 1997), the UNICONE approach (Eslami & Fellenius, 1997), and a method by Kajima Technical Research Institute, KTRI (Takesue, et al. 1998). Figure 3 shows a summary graph for the LCPC evaluation of side friction ($f_p$) in clays that relies on the value of cone tip resistance at any particular elevation along the pile sides. For the LCPC method, the unit end bearing resistance for the pile is evaluated as $q_b = k_c q_t$, where $k_c = 0.40$ for nondisplacement piles (drilled) and $k_c = 0.55$ for displacement piles (driven), and $q_t$ = cone tip resistance beneath the pile toe. For sands, see details given in Bustamante & Frank (1997).

The Unicone method (Figure 4) relies on a five-part zonal categorization that is determined by plotting effective cone resistance ($q_t - u^2$) vs. sleeve friction ($f_s$). In this method, the unit pile side friction is evaluated from $f_p = c_{se}(q_t-u^2)$ where the values of $c_{se}$ are assigned per zone: $z1$ (0.08), $z2$ (0.05), $z3$ (0.025), $z4$ (0.01), and $z5$ (0.004). For the unit end bearing resistance, the Unicone method takes: $q_b \approx (q_t-u^2)$ beneath the pile tip. Additional details are found at: www.fellenius.net

**Figure 3.** Side friction in clays for various pile types per the LCPC method for CPT.
For the KTRI method, the pile side friction is estimated from the scaling of the CPT sleeve friction up or down, depending upon the induced excess porewater pressures measured by the piezocone. Figure 5 depicts the relationships that were derived for a clays, mixed soils, and sands from load testing of drilled shafts and driven pilings.

![Graph](image)

Figure 4. Soil behavioral type for Unicone Pile Method using piezocone results. (Eslami & Fellenius, 1997)

![Graph](image)

Figure 5. Pile side friction from CPT $f_s$ and $Du$ per the KTRI method (after Takesue, et al. 1998).
For a rational (or indirect) approach to the pile analysis using CPT results, either an alpha or beta method can be used to evaluate the unit pile side friction couple with limit plasticity solution beneath the pile toe for unit end bearing resistance. Details are given by Kulhawy et al. (1983) and O’Neill & Reese (1999). Specifically, for clays, the overconsolidation ratio can be evaluated from:

\[
\text{OCR} = 0.33 Q_t
\]

(4)

where \(Q_t = \frac{(q_t-\sigma_{vo})}{\sigma_{vo}'}\) = normalized cone tip resistance. Then, the lateral stress coefficient \((K_0)\) for simple loading-unloading can be evaluated from:

\[
K_0 = (1 - \sin\phi') \text{OCR} \sin\phi'
\]

(5)

where \(\phi'\) = effective stress friction angle, best determined from drained triaxial compression tests or consolidated undrained triaxial compression tests with porewater pressure measurements. It is theoretically possible to evaluate \(\phi'\) using the normalized cone resistance \(Q_t\) and normalized porewater pressures, \(B_q = \frac{\Delta u_2}{(q_t-\sigma_{vo})}\), using a procedure outlined by the Norwegian Institute of Technology (Senneset, et al. 1989). In that approach, assuming that the effective cohesion intercept \(c' = 0\):

\[
\phi' \text{(deg)} = 29.5 \cdot B_q^{0.121} \left[0.256 + 0.336 \log(B_q) + \log(Q_t)\right]
\]

(6)

Finally, the unit side friction of pile foundations can be determined from a beta-method approach (O’Neill, 2001):

\[
f_p = C_m C_k K_0 \tan\phi' \sigma_{vo}'
\]

(7)

where \(C_m\) = modifier term for pile material: cast in place concrete (1.0), prestressed concrete (0.9), timber (0.8), rusted steel (0.7); \(C_k\) = modifier for pile installation: drilled (0.9), augered (1.0), and driven (1.1).

For drained pile end bearing, the bearing factors are given elsewhere (e.g., Vesic, 1977; Kulhawy et al. 1983). For undrained loading with no volume change, the unit end bearing resistance is obtained from:

\[
q_b = *N_c s_u
\]

(8)

where \(*N_c\) = limit plasticity bearing factor (= 9.33 for circular foundation) and the value of undrained shear strength obtained from:

\[
s_u = (\sin\phi'/2) \text{OCR}^\Lambda \sigma_{vo}'
\]

(9)

where \(\Lambda = 1 - \frac{C_s}{C_c} \approx 0.80\) is commonly found for many clays and silts of low sensitivity.
NONLINEAR SOIL STIFFNESS

Soil stiffness begins at the fundamental value \( G_{\text{max}} = \rho_t V_s^2 \) and softens to lesser values \( G \) as loads are increased. One simple algorithm for modulus reduction is a modified hyperbolic form (Fahey, 1998) whereby:

\[
\frac{G}{G_{\text{max}}} = 1 - \left(\frac{1}{FS}\right)^g
\]  

(10)

where \( FS = \frac{Q_{\text{ult}}}{Q} \) = calculated factor of safety and \( g = \) exponent parameter. Thus, as working loads \( Q \) increase toward capacity \( (Q_{\text{ult}}) \), the modulus reduces accordingly. For uncedmented and nonstructured soils, the parameter \( g \approx 0.3 \pm 0.1 \) for many different soils (Mayne, 2005). For the small-strain region \( (\nu = 0.2) \), the shear modulus \( G \) converts to Young's modulus \( E \) by the elasticity relationship:

\[
E = 2G(1 + \nu)
\]  

(11)

Of particular value in geotechnical site characterization is the seismic piezocone test (SCPTu) as it provides four separate readings with depth from a single sounding, including: tip resistance \( (q_t) \), sleeve friction \( (f_s) \), porewater pressures at either tip \( (u_1) \) or shoulder \( (u_2) \) positions, and shear wave velocity \( (V_s) \). The SCPTu data allow for pile capacity analyses by both direct and indirect methods.

CASE STUDY APPLICATIONS

The Randolph-type analytical pile coupled with CPT interpretative methods will be presented using three case studies involving axial load testing of augered and bored piles in stiff clay soils at: (a) University of Houston; (b) Texas A&M University; and (c) Calgary, Alberta.

University of Houston

The University of Houston is host to one of the primary national geotechnical experimentation sites in the USA (O'Neill, 2000). The site is underlain by the stiff clays of the Beaumont formation which in turn overlies the stiff sandy clays of the Montgomery Formation below depths of about 8 m. Details on the site and subsurface conditions are given by O'Neill et al. (1982), Mahar & O'Neill (1983), O'Neill & Yoon (1995), and O'Neill (2000). The clays are Pleistocene age geomaterials that have become quite overconsolidated due primarily to desiccation. Figure 6 shows a summary of \( q_t \) and \( f_s \) profiles from nine CPTs conducted by the Louisiana Transportation Research Center (Tumay, 1997). In addition, the results of shoulder position porewater pressures \( (u_2) \) from a representative piezocone sounding at the site are presented (O'Neil and Yoon, 1995). The negative porewater readings are indicative of fissured overconsolidated soils (Lunne, et al. 1997).
Figure 6. Summary of cone soundings in Beaumont clays at University of Houston site (CPTs from Tumay, 1997; CPTu data from O’Neill and Yoon, 1995; O’Neill, 2000).

Figure 7. Results of load tests on ACIP piles in Texas by Vipulanandhan et al. (2005).
The UH site has served as a testing ground for a number of purposes including load testing of single piles and pile groups. Of particular interest today is the load testing of an augered cast-in-place pile (ACIP) at the UH site that has been reported by O’Neill et al. (2002) and Vipulanandan et al (2005). Figure 7 shows the results of several axial load tests on ACIP piles in Texas. A total capacity of around 1700 KN is evident for the UH pile which as a constructed diameter of \( d = 0.456 \) m and length of 15.2 m. From the load transfer measurements shown in Figure 8, it can be detailed that the mobilized side friction occurs in two major strata: (a) from 0 to 8 m \( (f_{p1} = 44 \text{ kPa}) \) and (b) from 8 to 15.2 m \( (f_{p2} = 109 \text{ kPa}) \).

We can compare the estimated side frictions from the various CPT methods discussed previously. For this, Figure 9 shows the unit side frictions from the indirect (rational) method, KTRI, Unicone, and LCPC approaches. Also shown is a method intended for driven piles in clay that is detailed by Powell, et al. (2001) and labelled as the BRE-NGI method. It is clear that the measured side friction values fall within those bounded by the various CPT interpretative ranges. The authors generally find it prudent to use a number of different CPT methods and see how they compare or disagree amongst each other. An averaging of the methods seems to be warranted in this case. For tip capacity, the various methods gave: 552 kN (LCPC), 523 kN (limit plasticity), and 570 kN (BRE-NGI), which are all fairly close.

In order to derive a complete load-displacement-capacity curve, the initial stiffness from shear wave measurements obtained in crosshole tests (CHT) can be used. These were made by Prof. Ken Stokoe of Univ. Texas-Austin (O’Neill, 2000) and presented in Figure 10a. Also shown are some CPT correlative methods for \( V_s \) which show comparable values. Corresponding \( E_{\text{max}} \) values are given in Figure 10b, with a representative homogeneous value taken as 364 MPa.

![Figure 8. Measured axial load transfer distribution in ACIP pile at UH. (after O’Neill, et al. 2002)](image_url)
Figure 9. Pile side friction resistances at UH evaluated using several CPT methods

Figure 10. Measured shear wave velocities and $E_{\text{max}}$ profiles in stiff clays at UH site. (CHT data reported in O'Neill, 2000)
Combining (1), (10), and (11) provides a direct means for calculating nonlinear load-displacement-capacity curves for pile foundation subjected to axial compression loading. The resulting expression for top displacements becomes:

$$w_t = \frac{Q_t \cdot I_p}{d \cdot E_{\text{max}} [1 - (Q_t/Q_{\text{ult}})^{0.3}]}$$  (12)

For a rigid pile floating in same soil medium, the displacement influence factor is simply that given by (2). Otherwise, for rigid pile bearing on a stiffer stratum, the elastic solution is given in Figure 2. For the general case of a compressible pile, the reader is directed to Randolph & Wroth (1978, 1979) or Fleming et al. (1992), or alternate form given by Mayne & Schneider (2001). The spreadsheet solution is given in the following table. The graphical comparison of the measured load test results and those calculated using the elastic continuum pile and equivalent SCPTu data are shown in Figure 11. For this example, it may be concluded that: (1) the axial capacity is well-matched by the CPT methods, (2) the proper axial distribution of load shed to sides and base is realized, (3) the initial pile stiffness is correct due to the use of fundamental stiffness ($E_{\text{max}}$), and (4) the modified hyperbola nicely fakes an approximate nonlinear modulus reduction.

**Table 1. Calculated Nonlinear Load-Displacement Response for UH ACIP Pile**

<table>
<thead>
<tr>
<th>ACIP Pile, Univ. Houston</th>
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<tr>
<td><strong>Length</strong></td>
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<td><strong>Diam.</strong></td>
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<tr>
<td><strong>Ave. $E_{\text{max}}$</strong></td>
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<td><strong>Q_{\text{cap.}}</strong></td>
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<td><strong>$Q/Q_{\text{ult}} = 1/FS$</strong></td>
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<td>0.98</td>
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Texas A&M Clay Site

At the NGES clay at Texas A&M University (TAMU), top down load testing of a drilled shaft (Pile No. 7) with \( d = 0.915 \, \text{m} \) and \( L = 10.7 \) was reported by Briaud, et al. (2000). The foundation was constructed as a "perfect pile", thus follows the upper curve for LCPC Type IA piles. Results of a seismic cone test (SCPT-20) have been combined with a nearby type 1 piezocone (CPT\(_u\)-12) reported by LTRC (Tumay, 1997) to produce an equivalent SCPT\(_u\) that is presented in Figure 12, with excellent agreement among the common \( q_t \) and \( f_s \) readings from both soundings. As both the KTRI and UNICONE methods rely strictly on the use type 2 piezocone data, the midface \( u_1 \) readings cannot be used for side and/or base capacities. Thus, the calculations here have been solely made on the basis of the LCPC method. This gives a calculated unit side friction of \( f_p = 58 \, \text{kPa} \) along the shaft and a unit end bearing resistance of \( q_b = 2270 \, \text{kPa} \). Measured shear wave velocity data in the upper 10.5 m give a mean stiffness value of \( E_{\text{max}} = 231 \, \text{MPa} \), however the base modulus can be better represented by a lower value \( E_b = 148 \, \text{MPa} \) that can also be accommodated by elastic pile solutions (e.g., Mayne & Schneider, 2001). Figure 13 shows the measured load test performance as compared with the elastic continuum pile with parameter evaluations by SCPT\(_u\), indicating excellent results.
Figure 12. Composite SCPTu sounding at the TAMU Clay Site, College Station, TX (data from Tumay 1997).

Figure 13. Measured and SCPT-evaluated shaft response at TAMU clay site. (load test data from Briaud, et al. 2000)
**O-Cell Tests in Calgary Clay Till**

Drilled shaft foundations were selected for support of the building loads for the Foothills Medical Center (FMC) in Calgary, Alberta. The site is underlain by thin shallow fill and surficial sandy silt layers over a thick deposit of stiff to hard silty clay till. Index properties of the clay till include: water content ($w_n$) between 13 to 17%, liquid limit (LL) = 27%, plasticity index (PI) = 10%, and clay fraction (CF < 0.002 mm) varying between 5 to 22%. The site investigation program included soil borings with N-values from standard penetration test (SPT) ranging between 30 and 60 blows/0.3 m. A seismic piezocone test (SCPTu) performed at site gave the readings shown in Figure 14.

To confirm design capacities, a test shaft was built with a 14-m embedded length and diameter of 1.4 m with the top of the foundation located 8 m below grade. The shaft was outfitted with an O-cell at a mid-elevation position 4 m above its base. The O-cell is an ingenious means to load test both the side friction and end bearing resistances by using a high pressure hydraulic jack to force one segment upward simultaneously forcing a lower segment downward (O’Neill, et al. 1997; Osterberg, 2000; Fellenius, 2001). After the testing, the jack is grouted up and becomes part of the final foundation. The O-cell requires a minimum of space, as compared with traditional reaction frames or dead weight loading systems.

As the elastic continuum pile model was originally developed by adding the solution for a circular plate to that for a simple axial shaft, the two components can be easily separated for analysis of O-cell load tests (Mayne & Woeller, 2008). Figure 15 shows the simple analytical solution for the O-cell load test. This provides an excellent means for post-processing the O-cell results, since they can be re-combined in a rational manner to simulate the actual top down loading that is imparted by the building superstructure. The measured and calculated curves for the two shaft segments tested with the O-cell at the Calgary FMC site are presented in Figure 16.

![Figure 14. SCPTu and elevations for O-cell at Calgary test shaft.](image)
O-Cell Elastic Solution

Rigid pile shaft under upward loading

\[
\frac{P_1}{G_s r_0 w_1} = \frac{2\pi}{\xi_1} \frac{L_1}{r_{01}}
\]

Rigid pile under compression loading

\[
\frac{P_2}{G_s r_0 w_2} = \frac{4}{(1-\nu)\xi_2} + \frac{2\pi}{\xi_2} \frac{L_2}{r_{02}}
\]

P = applied force  \quad w = pile displacement
L = pile length  \quad \lambda = E_p/G_s = soil-pile stiffness ratio
r_0 = pile shaft radius  \quad \xi = G_{12}/G_{sb} \quad \text{(Note: floating pile: } \xi = 1)\nE_p = pile modulus  \quad G_{sb} = soil modulus below pile base/toe
G_s = soil shear modulus along sides
\nu = Poisson's ratio of soil  \quad \xi = \ln(r_m/r_0) = soil zone of influence
\]

\[
m = L\{0.25 + \xi [2.5 (1-\nu) - 0.25]\}
\]

Figure 15. Elastic continuum solution for O-cell load testing of drilled shafts.

Figure 16. Measured O-cell response and SCPTu curves for Calgary test shaft.
CONCLUSIONS

Elastic continuum theory provides a rational and practical framework for the evaluation of load-displacement and axial load-transfer response of deep foundations, including driven pilings, augered piles, and drilled shafts. Axial loads can be applied top down as with conventional reaction frames, or in opposing base vs. shaft segments as occurs during O-cell testing. Seismic piezocone penetration tests (SCPTu) provide a wealth of geotechnical data on the subsurface conditions as four independent readings ($q_c$, $f_s$, $u_2$, $V_s$) are taken with depth in the same sounding. This is economical and efficient for routine site characterization, as the results provide information on the geostatigraphy and the evaluation of geotechnical parameters, including stress state, strength, stiffness, and permeability. Taken together, the Randolph elastic pile model with SCPTu data permits the evaluation of the complete load-displacement-capacity response and axial distribution of loads during the analysis and design of deep foundations. Three case studies involving instrumented load tests at the University of Houston, Texas A&M, and a medical building in Calgary, Alberta are reviewed to illustrate the applicability of the approach.

ACKNOWLEDGMENTS

Thanks are extended to Professor C. Vipulanandam for assisting in providing load test data at the UH site and to Chris Hendry of Golder Associates for providing the Calgary O-cell results.

REFERENCES


