Vipulanandan Failure Model for Plain Concrete and Property Correlations

K. Mayooran and C. Vipulanandan, Ph.D., P.E.
Center for Innovative Grouting Material and Technology (CIGMAT)
Department of Civil and Environmental Engineering
University of Houston, Houston, Texas 77204-4003
E-mail: mkrishnathasan@uh.edu, cvipulanandan@uh.edu Phone: (713) 743-4278

Abstract: In this study, uniaxial, biaxial and triaxial test data of plain concrete were used to evaluate and verify the generalized failure criterion. Vipulanandan failure model was compared with the Drucker- Prager failure model. Based on the correlation coefficient and RMSE (Root mean square error), Vipulanandan failure model predicted the results better than the Drucker-Prager(D-P) model.

1. Introduction:

These are fracture mechanic, plastic, linear and nonlinear elastic, damage model, smeared and discrete cracking models to describe the failure of several types of materials. The fracture criterion described in terms of stress invariants can be taken as the perfect plastic yield surface (Arslan, 2007). A considerable amount of numerical work has been done using (1) the von Misses criterion (1924), (2) Drucker–Prager (D-P) criterion (1952) and (3) the Coulomb or modified Coulomb criterion (1900) (Chen, 1982). A number of investigators have proposed models for defining the failure behavior of concrete. One of the early models was proposed by Kupfer and Gerstle (1969) involved (separate) definitions of failure surface for biaxial tension, biaxial compression, and tension-compression zones (Salami & Desai, 1990). Kotsovos and Newman (1978), Ottosen, William and Warnke (1975, 1989, 1995), Lade, Chern et al. (1992), Dahl (1992), Imran and Pantazopoulou (1996), Li and Ansari (1999). and others have subsequently proposed models for describing the failure behavior of plain concrete (Sfer, et al., 2002). This project aims to compare the effects of the two parameters of a Druker–Prager type plasticity model and Vipulanandan failure model on its performance in predicting the behavior of failure surface of concrete and to identify the key characteristics a D–P plasticity model must possess in order to provide close predictions of test results. The assessment is focused on the D–P type concrete plasticity models because they have been widely used (Yua, et al., 2010) the conclusions reached for such models are also relevant to other plasticity models.

Mohr-Coulomb surface has corners on the hexogen which is not mathematically convenient. Drucker and Prager have smoothened the Mohr-Coulomb by simple modification of Von Mises criterion.
If we look at the Drucker and Prager model

$$\sqrt{J_2}\ \alpha\ I_1$$

I_1 increases linearly with principal stresses. But conceptually Material will show crushing behavior at high principal stresses. So, the $\sqrt{J_2}$ should start flattening after a certain I_1 value. Vipulanandan has modified the criterion to a three-parameter model to satisfy these mentioned conflicts. Finally, the modified concrete plasticity model is presented and verified with gathered data.

2. Objective:

Main objective was to compare the Vipulanandan failure model to the Drucker Prager model to predict the failure surface of plain concrete. Totally 42 data from (Imran & Pantazopoulou, 1996) were used for this investigation. The specific objectives are

1) Analyze the correlation of the both model with experimental points by using coefficient of correlation and Root mean square error.
2) Correlate the material parameters with the properties of concrete.

3. Models:

Drucker–Prager (D–P) type plasticity model

Drucker–Prager (D–P) criterion has been widely adopted for the modeling of confined concrete because of its simplicity (involving only two parameters) and its capability to capture shear strength increases as a result of hydrostatic pressure increases, which is a unique property of concrete under confinement.

$$\sqrt{J2} - \alpha I1 - K = 0 \qquad (3.1)$$

in which I1 is the hydrostatic component of the stress tensor (Stress invariant), J2 is deviatoric stress invariants and α and k are material constants.

Vipulanandan Failure model

The Vipulanandan failure model was developed to satisfy the condition in (Equation 3.3 and Equation 3.4). Based on the inspection of the test data the Vipulanandan failure relationship (Equation 3.2) was used to predict the concrete failure behavior.

$$\sqrt{J_2} = \tau_0 + \frac{I_1}{A+BI_1} \qquad (3.2)$$

Hence the conditions are as follows:

$$\sqrt{J_2} = \tau_0 \quad \text{When} \quad I_1 = 0 \qquad (3.3)$$

$$\sqrt{J_2}_{max} = \tau_0 + \frac{1}{B} \quad \text{When} \quad I1 \rightarrow \alpha \qquad (3.4)$$

Hence, this model has a limit on the maximum shear stress the concrete will tolerate at relatively high normal stress. It satisfies the Drucker and Prager model when B = 0 and shows Von Mises Criterion when A, B = 0.

4. Results and Discussion:

The Vipulanandan proposed failure surface was implemented into the constitutive model for concrete and it was verified against experimental results. Two additional data points were used in the study in addition to the gathered experimental data. Those are the pure shear condition and Direct tension condition. In this study, Vipulanandan Model parameter τo and Drucker Prager parameter K were determined for concrete with different unconfined compressive strengths by adding those two points and the other model parameters were determined by using the least square error method. The comparison of experimental and predicted values of $\sqrt{J2}$ (Deviatoric stress invariant) with I1(Normal Stress invariant) is illustrated in the Figure 2 for 10640 psi unconfined compressive strength samples. Another two parameter Drucker Prager model was used to model the same obtained experimental data and showed in the same $\sqrt{J2} - I1$ plane. Likewise, 3070 psi, 4150 psi, 6250 psi, 6870 psi and 9380 psi compressive strength concrete data were modelled and the error (RMSE) was compared in the

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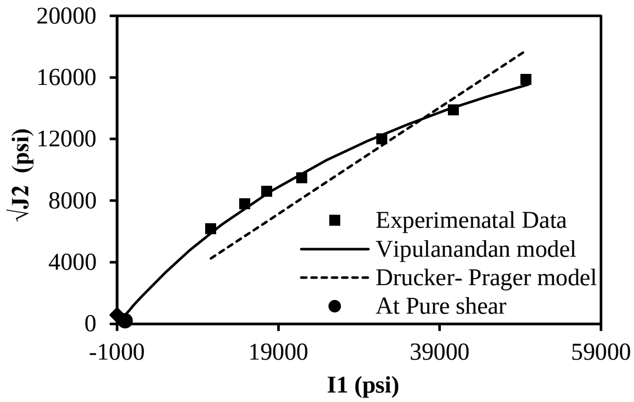

Figure 1: Failure surface of plain concrete with different strength of 10640 psi in√J2-I1 plane

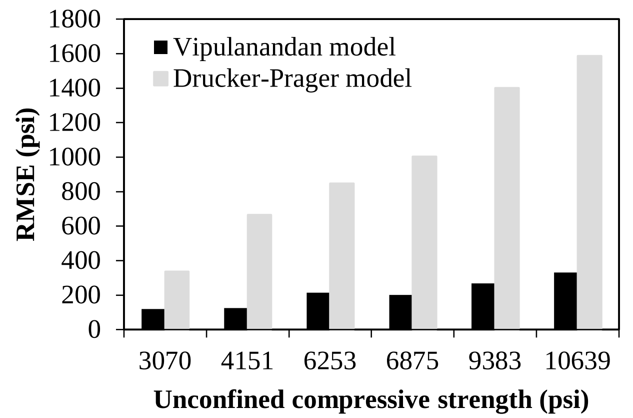

Figure 2: Compared errors (RMSE) of failure criteria for plain concrete

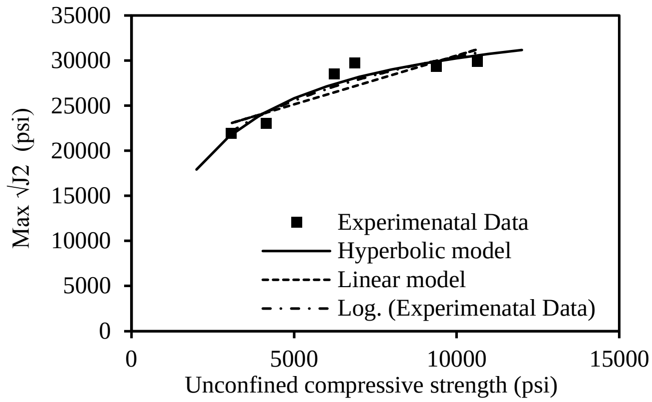

Figure 3: Variation of Max √J2 with unconfined compressive strength

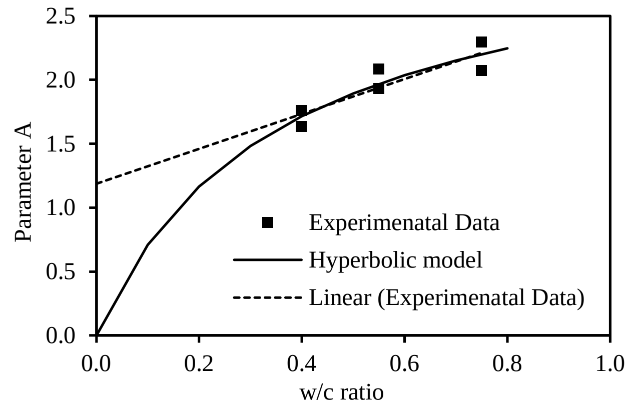

Figure 4: Variation of model Parameter A with w/c ratio

Shear tolerance (Max √J2) was calculated for all different compressive strength concrete and correlated with the unconfined compressive strength by using the hyperbolic, linear and log models. Correlation coefficient (R^2) was 0.90, 0.76 and 0.87 respectively. Figure 4 shows the correlation between the Parameter A and the water/cement ratio. Linear model was predicting the results with a R^2 of 0.8 and the R^2 of the hyperbolic equation was 0.83.

5. Conclusion:

In this study, two failure models were used to analyze results from the multiaxial load test on plain concrete.

- The Vipulanandan failure model strongly correlated the $\sqrt{J2}$ and I1 of 3070 psi, 4150 psi, 6250 psi, 6870 psi and 9380 psi and 10640 psi compressive strength concrete with coefficient of correlation (R^2) of 1.00, 1.00, 0.99, 1.00, 0.99 and 0.99 respectively.
- Vipulanandan model showed better correlation than the Drucker and Prager model on the prediction of failure surface.
- Hyperbolic model was better predicted the Max √J2 against the unconfined compressive strength with the R^2 of 0.9.
- Also, Hyperbolic model shown better correlation compare to linear in predicting the Parameter A by using the water/cement ratio.

6. Acknowledgements:

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