Equivalent Resistance of A 3-D Conductive Medium

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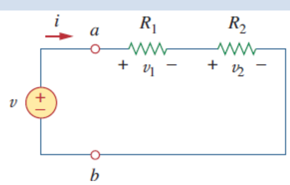
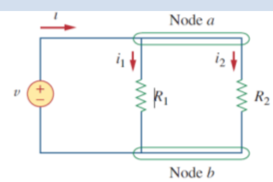
Abstract: Making the drilling fluids more sensing and measuring the resistance of drilling fluids could provide more information on the performance. Hence, in this study the equivalent resistance of a 3-D conductive block was analyzed using the principals of electricity. Stability of a wellbore during drilling depends on the drilling fluid characteristics which can be measured and improved by monitoring the resistance of drilling fluid. The anisotropic 3-D block was represented with three different orthogonal resistances ($R_x \geq R_y \geq R_z$). The maximum and minimum resistances were $R_x + R_y$ and R_z, respectively.

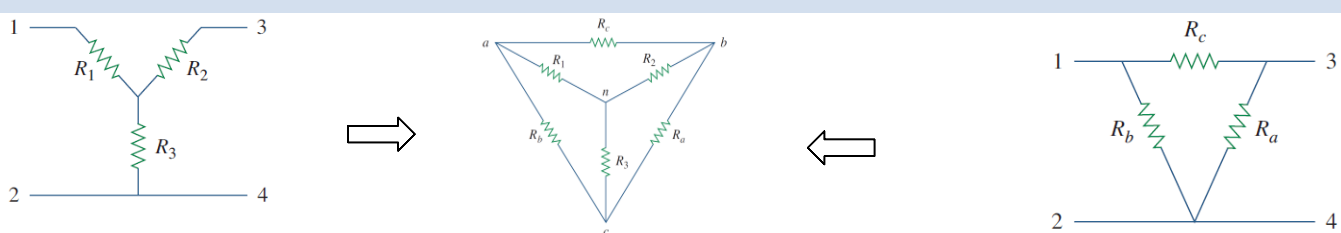
1. Introduction: Maintaining wellbore stability is a challenging problem in petroleum production. Drilling efficiency and also maintenance of the borehole stability will be improved by optimizing the drilling fluid parameters during drilling process. The mud's sensing capability provides the measurement of mud's parameters consist of density, viscosity, salt concentration and temperature which can be measured by using a sensitive material as drilling fluid.

2. Objective: The main objective was to quantify the evaluation of electrical resistance in different directions and also determine the maximum and minimum resistances.

3. Techniques and Methods: The technique includes combining resistors in series or parallel, and delta-to-wye or wye-to-delta transformations to determine the equivalent resistance of a circuit (Table 1,2 and 3). By determining the equivalent resistances in different directions, it will be possible to determine the maximum and minimum resistances.

Table 1. Methods for evaluating equivalent resistance in different network; (a) Series, (b) Parallel, (c) delta-to-wye and (d) wye-to-delta

(a) Two resistors in Series	*(b) Two resistors in Parallel*
	
$R_{eq} = R_1 + R_2$	$1/R_{eq} = 1/R_1 + 1/R_2$
Generally	***Generally***
$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$	$1/R_{eq} = 1/R_1 + 1/R_2 + \cdots + 1/R_N = \sum_{n=1}^{N} 1/R_n$
(c) Delta to Wye Conversion	*(d) Wye to Delta Conversion*

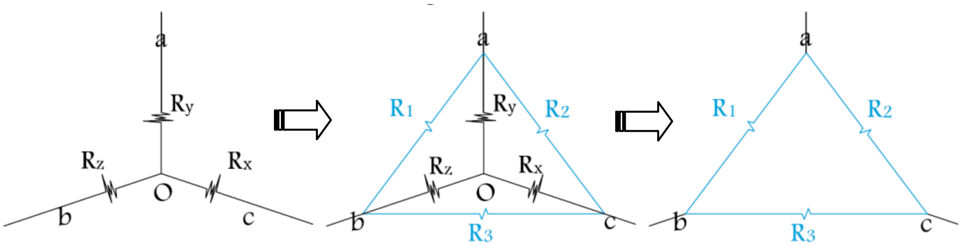


(c) Delta to Wye Conversion	*(d) Wye to Delta Conversion*
$R_1 = \frac{R_c R_b}{R_a + R_b + R_c}$	$R_a = \frac{R_1R_2 + R_3R_2 + R_1R_3}{R_1}$
$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$	$R_b = \frac{R_1R_2 + R_3R_2 + R_1R_3}{R_2}$
$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$	$R_c = \frac{R_1R_2 + R_3R_2 + R_1R_3}{R_3}$

4. A 3-D problem: The conductive medium was idealized as having three different orthogonal resistances located in 3 direction of xyz coordinate system have been considered. As mentioned equivalent resistivity theories including series, parallel and wye to delta conversion were used to simplify and find equivalent resistance. In order to achieve equivalent resistance, by applying Y to Δ conversion, point O is removed and R_x, R_y and R_z are replaced by R_1, R_2 and R_3. Then, the equivalent resistance is defined between each two points using parallel and series rules.

Table 2. Wye to Delta Conversion rule

$R_1 = (R_xR_y + R_xR_z + R_yR_z)/R_x$
$R_2 = (R_xR_y + R_xR_z + R_yR_z)/R_z$
$R_3 = (R_xR_y + R_xR_z + R_yR_z)/R_y$



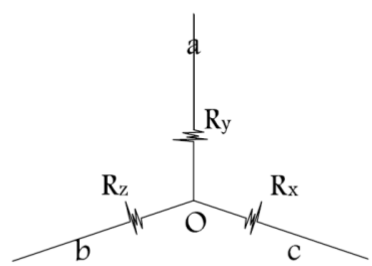
$$R_{eq(ab)}\{(R_2 + R_3) \parallel R_1\} = \frac{(R_2 + R_3) \times R_1}{R_1 + R_2 + R_3} \quad (1)$$

By substituting equations of R_1, R_2 and R_3: $R_{eq(ab)} = R_z + R_y$

Similarly: $R_{eq(ac)} = R_x + R_y$, $R_{eq(bc)} = R_x + R_z$

Table 3. Equivalent resistance in different direction (assume: $R_x \geq R_y \geq R_z$)

$R_{eq(ab)} = R_y + R_z$
$R_{eq(ac)} = R_x + R_y$
$R_{eq(bc)} = R_x + R_z$
$R_{eq(max)} = R_{eq(ac)}$
$R_{min} = R_z$



5. Conclusion: Based on this study the equivalent electrical resistance in the 3 directions was determined simply by means of basic laws in electrical resistance network. While evaluating equivalent resistance between each two nodes, the third resistance in each analysis does not affect the equivalent resistance because the third resistance was not located in a closed circuit.

6. Acknowledgements: This study was supported by the Center for Innovative Grouting Materials and Technology (CIGMAT), University of Houston, Houston, Texas with funding from the Ultra Deepwater Program DOE/NETL/RPSEA (Project No. 10121-4501-01).

7. References

http://highered.mcgraw-hill.com/sites/dl/free/0073380571/938343/Chapter_2.pdf.