Modeling Piezoresistive Behavior of a Polymeric Material

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Abstract: By improving the material properties, structural element can be made smarter to act as sensor, which is self-sensing depending on material's intrinsic properties. If the electrical resistivity of those composites change under applied stress or strain, the material is referred to as piezoresistive. Electro-mechanical constitutive relationships given in literature are of interest to model the piezoresistive behavior of such polymeric materials. In this study, different constitutive relationships based on the principle of continuum mechanics and percolation theory were analyzed. Piezoresistive behavior of a polymeric composite (denoted as LL) was modeled and validated using experimental results.

1. Introduction

Polymer composites are used in light weight structures such as aerospace and automotive mainly because of their properties such as rapid setting, high strength, good adhesive property, and durability. Piezoresistivity of polymeric composites has been proven to be a good sensing property and it can be used to self-sense stress/strain, sense damage and thermoelectric properties and monitor health of the structure and more. Constitutive modeling of piezoresistive behavior is important since it is used for sensing. Constitutive relationships proposed by Sett (2003) and Park et al. (2002) both of which were electro-mechanical constitutive models based on the principle of continuum mechanics have been selected in this study. In these, fractional change in electrical resistivity or resistance was related to stress or strain by piezoresistivity coefficient and elasto-resistance tensor (known as gage factor).

2. Objectives

The main objectives of this study were to analyze different constitutive relationships to model the piezoresistive behavior of a polymeric composite based on those relationships and also to quantify the piezoresistive parameters for the composites.

3. Piezoresistive models

As derived by Sett (2003), the change in resistivity of linear elastic, isotropic, homogeneous composite under applied stress or strain can be represented as follows

$$\left(\frac{\Delta\rho}{\rho_0}\right)_i = \boldsymbol{\Pi}_{ijk}\boldsymbol{\Delta\sigma}_{jk} = \boldsymbol{\Pi}_{ijk}\boldsymbol{C}_{jkmn}\boldsymbol{\Delta\varepsilon}_{mn} = \boldsymbol{M}_{ijk}\boldsymbol{\Delta\varepsilon}_{jk} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

where C_{jkmn} is the elasticity matrix and the tensor M_{ijk} is the elasto-resistance tensor known as the gage factor which signifies the sensitivity of change in resistivity measurement to strain measurement. The piezoresistivity co-efficient Π_{ijk} , which relates the specific change in electrical resistivity to the change in stress tensor was defined in terms of fiber properties (E^f, μ^f and B^f) and composite properties (E^c, μ^c, B^c), along with fiber volume fraction (ϕ), critical volume fraction (ϕ_{crit}) and parameter Z which represents the rate of change of microstructure. Here E is Young's modulus, μ is Poisson's ratio and B is the shear parameter.

The model proposed by Park et al. (2002) is given in Eqn. (2), where composite was considered as a DC circuit having a parallel array of electrical resistors in which breakage of fibers is analogous to loss of resistors.

$$\left(\frac{\Delta R_{total}}{R_{total,0}}\right) = \frac{(1+\alpha\varepsilon)}{\exp\left[-\left(\frac{\delta_{ec}}{L_0}\right)\left(\frac{E_f\varepsilon}{\sigma_0}\right)^m\right]} - 1 \quad \text{……………..} \; (2)$$

Here R is the electrical resistance, δ_{ec} is the 'electrical ineffective length', the length over which a broken fiber does not carry electrical current, L_o is the fiber length, α is the gage factor and σ_o and m are Weibull scale factor and shape factor respectively.

4. Results and Analysis

Above mentioned models were used to predict the piezoresistive behavior of a 1.5*3.5 inch cylindrical LL specimen which was tested under uniaxial compressive strength. Strain and electrical resistance were measured with applied stress. Stress-Resistivity and Strain-Resistivity predictions according to Sett's model and Park et al. model are shown in Figure 1 along with experimental data. As seen in the figure, both models predicted the piezoresistive behavior reasonably well. Mean squared error for Sett's model and Park et al.'s model were 4.86 and 7.08 respectively.

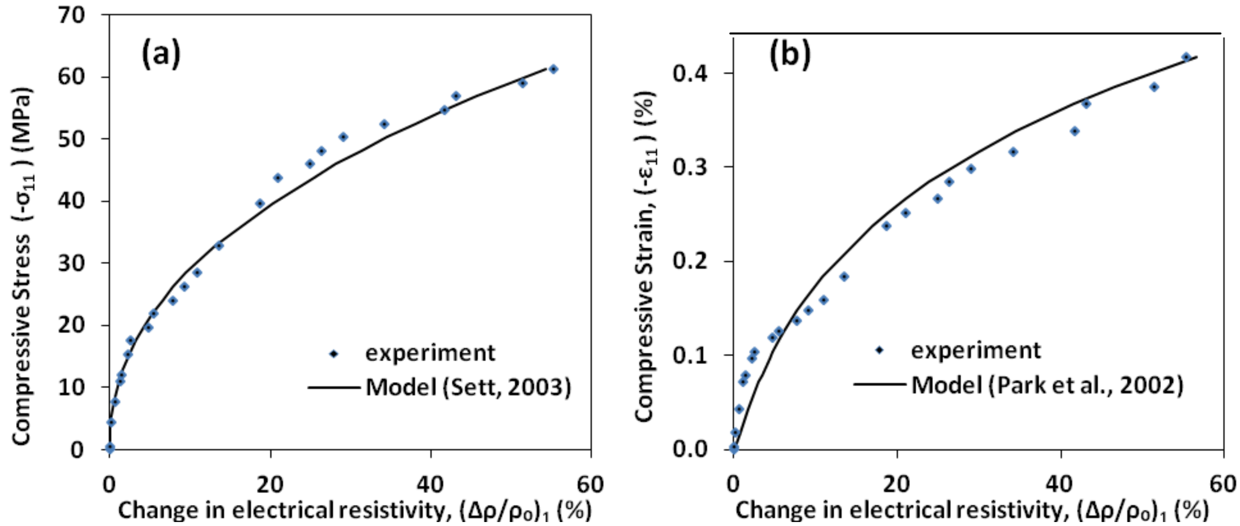

Figure 1: Modeling the Piezoresistive behavior of LL; (a) Stress vs. resistivity using Sett's (2003) model, (b) Strain vs. resistivity using Park et al. (2002) model

5. Conclusions

Selected analytical models predicted the piezoresistive behavior of LL well. It can be concluded that those two electromechanical models are valid for LL material.

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7. Reference

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